Notes on evolution of axiomatic characterization of the Tsallis entropy

Edin Mulalić¹, Miomir Stanković², Radomir Stanković³

¹Mathematical Institute of the Serbian Academy of Sciences and Arts, Kneza Mihaila 36, 11000 Belgrade, Serbia
²University of Niš, Faculty of Occupational Safety, Čarnojevića 10a, 18000 Niš, Serbia
³University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Niš, Serbia
E-mail: ¹edinmulalic@yahoo.com, ²miomir.stankovic@gmail.com, ³radomir.stankovic@gmail.com

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Summary

The Tsallis entropy was proposed as a possible generalization of the standard Boltzmann-Gibbs-Shannon (BGS) entropy as a concept aimed at efficient characterisation of non-extensive complex systems. Ever since its introduction [1], it has been successfully applied in various fields [2]. In parallel, there have been numerous attempts to provide its formal derivation from an axiomatic foundation, for example [3, 4, 5, 6, 7, 8, 9]. Since the Tsallis entropy can be considered as a one-parameter generalization of the Shannon entropy in the sense that it reduces to the Shannon entropy in limiting case, one way to attack the problem of its axiomatization is by generalizing one of axiomatic systems developed for the Shannon entropy. However, it is not always obvious which is the most appropriate or natural way to do it. For example, different generalizations of the Shannon-Khinchin axioms have been proposed by at least three researchers [3, 4, 6]. Various other approaches to the problem of axiomatization of the Tsallis entropy have been explored and the topic continues to be the subject of debate in the scientific community. The debate is not constrained to advantages and disadvantages of a particular axiomatization, but questions about completeness and correctness also have been raised. Given the sheer number of the proposed axiomatizations, as well as mentioned issues, it is not an easy task to comprehend the current state of this topic. This is not surprising considering the fact that the Tsallis entropy itself has been rediscovered over times, independently by several researchers, as noted by Tsallis in [10]. In this talk, we present a brief overview of a class of axiomatic systems purposely developed for the characterization of the Tsallis entropy and investigate motivation for introducing each of them. We explore relationships among them, and show how one axiomatic system led to another, building in this way a chronological map of reappearances of definitions the Tsallis entropy and its various characterizations in scientific literature. In addition, we will discuss some of the existing issues and propose possible solutions, which can be viewed as a contribution to the problem of axiomatization of the Tsallis entropy [6, 7, 11]. In analysis of various axiomatic systems, certain concepts which require special attention and discussion are observed as particularly important. Accordingly, in these discussions we emphasize the role of the following concepts:

- **Additivity** – In which way the entropy of a composite system can be expressed in terms of entropies of its subsystems? So far, various types of additivity have been utilized, for example, the pseudo-additivity, generalized Shannon additivity and generalized Fadeev additivity.

- **Dependency** – How dependency of two systems affects their composition? Some axiomatizations exploit composition of independent systems, while others do not make such restrictions.

- **Trace-form** – Is the trace-form property taken as an assumption (an axiom) or does it follow as a consequence?

References


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‡The third author is supported by the Ministry of Education, Science and Technology of Republic of Serbia, Grant No. III44006


